Starter

In the binomial expansion of
where ,
the sum of the coefficients of
and is times the coefficient
of .

Find the value of .

$$\square\square\square\square = \frac{\square!}{\square!\left(\square - \square\right)!}$$

0 Statistical hypothesis testing

01

Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, p-value; extend to correlation coefficients as measures of how close data points lie to a straight line and be able to interpret a given correlation coefficient using a given p-value or critical value (calculation of correlation coefficients is excluded).

Students should:

- recognise whether a given context requires the use of a 1-tail or 2-tail test and understand the difference between them
- be able to state appropriate null and alternative hypotheses to test a population proportion in a given context and know that the null hypothesis always contains the equality sign
- understand that the significance level of a test is the probability of rejecting a correct null hypothesis in error
- be able to find the test statistic, which is the observed number of outcomes of the event
- be able to find the critical region for a 1-tail test, or the critical regions for a 2-tail test, supporting the choice of values in such regions with appropriate binomial probabilities
- know that the critical region consists of the critical values for the test and that if the test statistic lies in the critical region then the null hypothesis is rejected
- know that the acceptance region is the range of possible values, that the discrete random variable can take, that do not lie in the critical region and that if the test statistic lies in the acceptance region that this will lead to the acceptance of the null hypothesis
- be able to use the given p-value corresponding to the test statistic or the given critical value(s), for the relevant significance level of the test, to decide whether to accept or reject the null hypothesis; understand that the p-value should be compared to a binomial distribution critical region with probability equal to or less than the significance level
- know that the precise definition of a p-value in a 2-tailed test varies. It can be defined as the probability calculated from the test statistic or twice that value. In order to circumvent this difficulty, questions will not be asked in which students are required to state the p-value for such a test
- be able to interpret a conclusion in context.

0 Statistical hypothesis testing

Notes

- The conclusion of a hypothesis test is an inference based on evidence and thus students must indicate that there is no certainty in their conclusions. Using the phrase “sufficient evidence to suggest” (qualified with a “not” as applicable) would be a good standard to adopt. There is nothing to be gained by trying to write a conclusion creatively. The final concluding statement of a hypothesis test should always relate back to the context.
- In cases where the null hypothesis is not rejected we would allow an inference of “Do not reject H0” or “Accept H0.” Statistical purists will prefer the former.

11.1 Hypothesis Testing

A **hypothesis** is a statement that you want to **test**.

We want to know whether a statement about a whole population is believable or not.

11.1 Hypothesis Testing

Null and Alternative Hypotheses

A hypothesis test (or significance test) is a method of testing a claim about a **population parameter** (e.g. mean (), variance (), proportion () etc...) using **observed data** from a **sample**.

11.1 Hypothesis Testing

Null and Alternative Hypotheses

The **Null Hypothesis** () is a statement about a population parameter (our data may allow us to reject this statement).

It is always a **specific** value (e.g.).

We can get two possible outcomes from a hypothesis test:

1. ***Reject*** : data provides **sufficient** evidence to suggest that the null hypothesis is **untrue**.
2. ***Do not reject*** : data does **not** provide

11.1 Hypothesis Testing

Null and Alternative Hypotheses

The **Alternative Hypothesis** () is what we conclude if we end up rejecting (what are we rejecting in favour of?).

A one-tailed specifies whether the parameter we are investigating is greater than or less than the value in (e.g.)

A two-tailed H_1 just says that the parameter is not equal to the value in H_0 (e.g.)

11.1 Hypothesis Testing

Example 1

The average probability over the course of a day that a customer entering a particular post office has to queue for more than 2 minutes is 0.6.

The manager wants to test whether the probability is different between the hours of 1pm and 2pm.

a) Write down a suitable null hypothesis

probability of having to queue for more than 2 minutes

11.1 Hypothesis Testing

Example 1

The average probability over the course of a day that a customer entering a particular post office has to queue for more than 2 minutes is 0.6.

The manager wants to test whether the probability is different between the hours of 1pm and 2pm.

b) Write down a suitable alternative hypothesis

11.1 Hypothesis Testing

Example 1

The average probability over the course of a day that a customer entering a particular post office has to queue for more than 2 minutes is 0.6.

The manager wants to test whether the probability is different between the hours of 1pm and 2pm.

c) State whether this is a one or two-tailed test

Two-tailed

11.1 Hypothesis Testing

To set up a Hypothesis Test (for a binomial distribution):

1. Define the population parameter **in context**. For a binomial distribution it is always p, a **probability** of success **or** a **proportion** of the population.

2. Write the null and alternative hypotheses:
   (is a constant)
   or (1-tailed test)
   (2-tailed test)

11.1 Hypothesis Testing

To set up a Hypothesis Test (for a binomial distribution):

3. State the **test statistic**,
   A test statistic for a hypothesis test is a statistic calculated from **sample data**, which is used to decide whether or not to reject .
   For a binomial distribution, is always the ‘no. of successes’ in the sample.

4. Write down the **probability distribution** of under the null hypothesis (assuming H_0 is correct):

11.1 Hypothesis Testing

Example 2

Cleo wants to test whether a coin is more likely to land on heads than tails. She plans to flip it 15 times and record the results. Write down suitable null and alternative hypotheses. Define the test statistic, X, and give its probability distribution under the null hypothesis.

Let p be the probability of the coin landing on heads.

(one-tailed test)

11.1 Hypothesis Testing

General Method

1. Define the population parameter in context (Let be…).
2. State the null hypothesis ().
3. State the alternative hypothesis ().
4. State the test statistic (Let be the number of…).
5. Write the probability distribution of (Under ,).
6. *State the significance level ().*
7. *Test for significance (-value) OR find the critical region.*
8. *Write conclusion in context (is there sufficient evidence to reject .)*

Ex 1.1 & 2.1